First, we know that atmospheric pressure is acting on the free surface.

By common sense, we know that a fluid will not undergo a vertical jump without any external force, i.e., there are vertical forces balancing the atmospheric pressure and weight of the fluid.

\[ \text{No vertical motion} \]

Since the gravity acting in direction \(-g\), \( \frac{dp}{dy} = -pg \)

i.e., pressure along BC has a gradient of \( pg \)

\[ \text{For any point D in between B and C, the pressure is given as } \ P_D = P_B + pg \gamma_D = P_A + pg \gamma_D \]

where \( \gamma_D \) is the distance of D from the surface.
So, \[ P_a + \rho g y_0 \]

Since on the left, the only balancing force is the atmospheric pressure (NO shear):

\[ Pa + \rho g y_0 \]

\[ \therefore Pa \sin \theta (\ell_{AB}) < (Pa + \rho g y_0)(\ell_{BC}) \]

Pressure Area \( AB \) Pressure Area \( BC \)

Horizontal forces are unbalance, so the fluid will be flowing to the left.

Attempted to explain +1
Developed argument +2/3
Realized that pressure change in fluid +4/15
Given: \[ u = B \frac{\Delta p}{M} (r_0^2 - r^2) \]

Find dimension of \( B \):

\[
[W] = \left[ \frac{\text{L}}{\text{T}^2} \right], \quad [\Delta p] = \left[ \frac{\text{M}}{\text{LT}^2} \right], \quad [M] = \left[ \frac{\text{M}}{\text{LT}} \right], \quad [r_0^2] = [r] = [\text{L}^2]
\]

\[
\left[ \frac{\text{L}}{\text{T}^2} \right] = [B] \left[ \frac{\text{M}}{\text{LT}^2} \right] \left[ \frac{\text{LT}}{\text{M}} \right] [\text{L}^2]
\]

\[
\left[ \frac{\text{L}}{\text{T}^2} \right] = [B] \left[ \frac{\text{M}}{\text{LT}^2} \right] \left[ \frac{\text{LT}}{\text{M}} \right] [\text{L}^2]
\]

\[
[B] = \left[ \frac{\text{L}^2}{\text{M}} \right] \quad \therefore \quad \text{B has a unit of m}^{-1}
\]

**Conceal Answer with steps +5**

**Correct Answer with no step +2**

**Incorrect Answer with step +1**
To verify \( \mu \), \( \gamma \), and \( \nu \) can form a dimensionless quantity.


Assign \( [\frac{M}{LT}]^a [\frac{M}{T^2}]^b [\frac{L}{T}]^c = [1] \)

Assign \( a = 1 \)

\( [\frac{M}{LT}]^a [\frac{M}{T^2}]^b [\frac{L}{T}]^c = [1] \)

We can set up 3 simultaneous equations by equating power of \([M],[L],[T]\), and \([T]\)

\([M]\) : \(a + b = 0\) \quad \Rightarrow \quad a = -b
\([L]\) : \(-a + c = 0\) \quad \Rightarrow \quad a = c
\([T]\) : \(-a - 2b + c = 0\) \quad \Rightarrow \quad b = -c

Assign \( a = 1 \)

\( \Rightarrow b = -1, \quad c = 1 \)

\( \therefore [\mu] \cdot [\gamma]^{-1} \cdot [\nu] = [1] \)

\( \therefore \frac{\mu \nu}{\gamma} \) or \( \frac{\nu}{\mu \nu} \) is dimensionless

b) By Table A-3

\( T = 20^\circ C, \quad U = 3.5 \text{ cm/s}, \quad \mu = 0.001 \text{ kg/m.s}, \quad \gamma = 0.0728 \text{ kN/m} \)

\( \frac{\mu U}{\gamma} = \frac{(0.001)(0.035)}{0.0728} \)

\( = 0.00487 \quad \text{Ans} + 2 \quad \rightarrow \text{These groupings are called capillary number.} \)
1.37  Ideal gas \( M = 44 \text{ g/mol}, C_v = 610 \text{ J/kgK} \), p.5

Find a) Specific Heat Ratio \( k \), b) Speed of Sound at 100°C

a) \( R = \frac{p}{M} \)

\[
R = \frac{8.314 \ \text{N.m/mol.K}}{44 \ \text{g/mol}} = \frac{8.314 \ \text{N.m/mol.K}}{0.044 \ \text{kg/mol}} = 189 \ \text{N.m/kg.K}
\]

\( C_p = C_v + R \)

\[
C_p = 610 + 189 = 799 \ \text{J/kg.K}
\]

\[
k = \frac{C_p}{C_v} = \frac{799}{610} = 1.31 + 3 \quad \text{Find } k \text{ by } k = \frac{C_p}{C_v} + 1 \text{ exact}
\]

b) \( a = \sqrt{kRT} \)

\[
a = \sqrt{(189)(184)(273+100)} = 304 \text{ m/s} + 2
\]

1.45  FBD:

Consider \( x \)-component only

For \( a_x = 0 \),

\[
W \sin \theta = \tau A
\]

\[
W \sin \theta = \mu \frac{h}{A}
\]

\[
V_{\text{terminal}} = \frac{h \frac{W \sin \theta}{\mu A}} + 3
\]

By Table A-3, \( \mu = 0.29 \text{ kg/m.s} \)

Given \( A = 35 \text{ cm}^2, \theta = 15^\circ, h = 0.001 \text{ m, } m = 6 \text{ kg} \)

\[
V_{\text{terminal}} = \frac{(0.001)(35)(0.29)}{(0.29)(0.0003)} = 15 \text{ m/s}
\]
Given $M = \frac{\pi \rho_0^4 \Delta p}{8 L Q}$, $\rho_0 = 2\text{mm}$, $L = 25\text{cm}$

<table>
<thead>
<tr>
<th>$Q$ (m$^3$/hr)</th>
<th>0.36</th>
<th>0.72</th>
<th>1.04</th>
<th>1.44</th>
<th>1.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta p$ (kPa)</td>
<td>159</td>
<td>318</td>
<td>477</td>
<td>1274</td>
<td>1851</td>
</tr>
</tbody>
</table>

For $Q = 0.36$ m$^3$/hr, $\Delta p = 159$ kPa

Apply formula given

$$Q = 0.36 \text{ m}^3/\text{hr} \left( \frac{1 \text{ hr}}{3600 \text{ s}} \right) = 1 \times 10^{-4} \text{ m}^3/\text{s}$$

$$M_1 = \frac{\pi (0.002)^4 (159000 \text{ Pa})}{8 (0.25)^3 (1 \times 10^{-4} \text{ m}^3/\text{s})} = 0.03996 \frac{\text{N \cdot s}}{\text{m}^2} + 2$$

Similarly,

$$M_2 = \frac{\pi (0.002)^4 (218000)}{8 (0.25)^3 (2 \times 10^{-4})} = 0.03996 \frac{\text{N \cdot s}}{\text{m}^2}$$

$$M_3 = \frac{\pi (0.002)^4 (477000)}{8 (0.25)^3 (3 \times 10^{-4})} = 0.03996 \frac{\text{N \cdot s}}{\text{m}^2} + 2$$

$$M_4 = \frac{\pi (0.002)^4 (1274000)}{8 (0.25)^3 (4 \times 10^{-4})} = 0.08004 \frac{\text{N \cdot s}}{\text{m}^2} \rightarrow \text{Incorrect}$$

$$M_5 = \frac{\pi (0.002)^4 (1851000)}{8 (0.25)^3 (5 \times 10^{-4})} = 0.09304 \frac{\text{N \cdot s}}{\text{m}^2} \rightarrow \text{Incorrect}$$

$$M = 0.03996 \frac{\text{N \cdot s}}{\text{m}^2}$$

Last two points are incorrect due to high Reynolds Number, which results in turbulent flow
Given \( T = 20^\circ C, \ Pa = 131 \text{ kPa}, \ C_{a\text{crit}} = 0.25 \)

Check what \( V \) is for cavitation to occur.

\[
C_a = \frac{Pa - Pa}{\frac{1}{2} \rho V^2} \quad \text{By Table A-5}
\]

\[
R_p = 2.337 \text{ kPa at } T = 20^\circ C
\]

By Table A-1

\[
\rho = 998 \text{ kg/m}^3 \text{ at } T = 20^\circ C
\]

\[
C_a = \frac{(131 - 2.337)(1000)}{\frac{1}{2}(998)(V^2)} = 0.25
\]

Solving for \( V \), \( V = 32.114 \text{ m/s} \)

For \( T = 5^\circ C, \ V = 30 \text{ m/s} \)

Interpolating Table A-5 and Table A-1

\[
R_p = 0.611 + \frac{1.227 - 0.611}{10 - 0} (5) = 0.919 \text{ kPa}
\]

\[
\rho = 1000 \text{ kg/m}^3
\]

\[
C_a = \frac{(131 - 0.919)(1000)}{\frac{1}{2}(1000)(30)^2} = 0.289
\]

.: For \( C_a > C_{a\text{crit}} \), cavitation will not occur.

At \( T = 5^\circ C, \ V = 30 \text{ m/s} \)

Cavitation will not occur
Given \( 555 \text{ mi/h} \), \( M = 0.8 \)

Find Altitude

\[
1 \text{ m/h} = 0.44704 \text{ m/s}
\]

\[
555 \text{ m/h} = (555 \text{ m/h})(0.44704 \text{ m/s}) = 248.1072 \text{ m/s}
\]

\[
V = Ma
\]

\[
a = \frac{V}{M} = \frac{248.1072}{0.8} = 310.1 \text{ m/s}
\]

By table A-6, \( z \approx 7500 \text{ m} \) for \( a = 310.1 \text{ m/s} \)

The altitude is \( 7500 \text{ m} \)

---

2.5

Given altitude = \( 5300 \text{ ft} \)

\( P_a = 83 \text{ kPa} \), \( P_b = 105 \text{ kPa} \)

\( 5300 \text{ ft} = 1615.44 \text{ m} \)

By table A-6, perform interpolation

\[
P = 84565 + \frac{79500 - 84565}{2000 - 1500} (1615.44 - 1500)
\]

\[
= 83.40 \text{ kPa}
\]

\( P_{gage} = P - P_{atm} \)

\( P_{vacuum} = P_{atm} - P \)

\[
\begin{align*}
P_{gage} &= 83 \text{ kPa} - 83.40 \text{ kPa} = -0.4 \text{ kPa} \\
P_{gage} &= 105 \text{ kPa} - 83.40 \text{ kPa} = 21.4 \text{ kPa}
\end{align*}
\]

\( \text{Pa vacuum} = -0.4 \text{ kPa} \)

\( \text{Pa vacuum} = -21.4 \text{ kPa} \)

+2 for knowing definition of \( P_{gage} \), \( P_{vacuum} \)

+2 for correct answer
Given altitude = 12000 ft, $T = 20^\circ C$

Find liquid rise in methanol barometer

\[ 12000 \text{ ft} = 3657.6 \text{ m} \]

Interpolating Table A-6

\[ \text{Path} = 65759 + \frac{(61633 - 65759)}{(4000 - 3500)} \cdot (3657.6 - 3500) \]

\[ = 64458 \text{ Pa} + 2 \]

For a methanol barometer:

\[ \text{Path} - \text{Pv} = \rho_{\text{methanol}} g h \]

By Table A-3, $\rho_{\text{methanol}} = 791 \text{ kg/m}^3$, $\text{Pv} = 13400 \text{ Pa} + 1$

\[ \frac{64458 - 13400}{791} \text{ (g) } h \]

\[ h = 6.58 \text{ m} + 1 \]

Note: We usually don't consider vapor pressure for mercury barometer because $\text{Pv}_{\text{Hg}} = 1.1 \times 10^{-3} \text{ Pa} \sim 0$
Given: $T = 20^\circ C$, $P_a = 1900 \text{ lb/ft}^2$

Find pressure at B, C, D

- By Table A-1, at $20^\circ C$
  \[ \rho_{H_2O} = 998 \text{ kg/m}^3 \]
  \[ \gamma_{H_2O} = 9790.38 \text{ N/m}^3 = 62.32 \text{ lb/ft}^2 \]

- By Table A-4
  \[ \rho_{gas} = 11.8 \text{ N/m}^3 = 0.07512 \text{ lb/ft}^3 \]

  \[ \Rightarrow \text{Insignificant when compared to } \gamma_{H_2O} \]

\[ \frac{dp}{dz} = -\gamma_{H_2O}, \quad \Delta p = -\gamma_{H_2O} (\Delta z) \]

\[ P_B - P_A = -(62.32)(z_B - z_A) = -62.32 \text{ lb/ft}^2 \]

\[ P_B = 1900 - 62.32 = 1838 \text{ lb/ft}^2 \]

\[ P_D - P_A = -(62.32)(z_D - z_A) = -(62.32)(-5) = 311.6 \text{ lb/ft}^2 \]

\[ P_D = 1900 + 311.6 = 2212 \text{ lb/ft}^2 \]

\[ P_D - P_C = -(62.32)(z_D - z_C) = -(62.32)(-2) = 124.64 \text{ lb/ft}^2 \]

\[ P_C = (124.64 - 2212) \]

\[ = 2087 \text{ lb/ft}^2 \]

Steps + 2

Answer + 3
Given \( M = 0.82 \), altitude = 24000 ft

- Find plane's velocity in \text{m/s}
- Find standard density at that altitude.

\[
24000 \text{ ft} = 7315.2 \text{ m}
\]

By Table A-6, perform interpolation for 7315.2 m,

\[
a = 312.3 + \frac{312.3 - 310.2}{7000 - 7500} (7315.2 - 7000)
\]

\[
= 310.97 \text{ m/s} \approx 311 \text{ m/s} + 3
\]

For standard density, by Table A-6, doing interpolation for 7315.2 m

\[
p = 0.5893 + \frac{0.5893 - 0.5564}{7000 - 7500} (7315.2 - 7000)
\]

\[
= 0.5686 \text{ kg/m}^3 + 2
\]
Find total pressure drop
Determine which part the manometer reads
Explain why.

\[ P_2 - P_1 = -\gamma (z_2 - z_1) \]

By Table A-3,
\[ \rho_{H_2O} = 9.98 \text{ kN/m}^3, \quad \rho_{Hg} = 13550 \text{ kN/m}^3 \]
\[ \gamma_{H_2O} = 0.098 \text{ kN/m}, \quad \gamma_{Hg} = 1329.25 \text{ kN/m} \]
\[ = 63.23 \text{ lbft/ft}^2 = 846 \text{ lbft/ft}^3 \]

\[ P_1 + (63.23)(\frac{6}{12} + h + 5\sin 45^\circ) = P_2 + 63.23h + 846 \left( \frac{6}{12} \right) \]

\[ P_1 - P_2 = 846 \left( \frac{6}{12} \right) - 63.23 \left( \frac{6}{12} \right) - (63.23)(5\sin 45^\circ) \]

\[ = 391.4 - 220.0 \]

\[ = 171 \text{ lbft/ft}^2 + 3 \]

Gravity Head is defined as pressure drop due to change in altitude:

Gravity Head = \(-\gamma h\)

Gravity Head = \(-\gamma_{H_2O} (z_1 - z_2) = -220.0 \text{ lbft/ft}^2 \)

There are only frictional loss and gravity head affecting pressure.

The term \( 846 \left( \frac{6}{12} \right) - 63.23 \left( \frac{6}{12} \right) \) can be assumed to be frictional loss.

The manometer reads: \( 846 \left( \frac{6}{12} \right) - 63.23 \left( \frac{6}{12} \right) = 391.4 \text{ lbft/ft}^2 \)

Only frictional loss can be read. +1 Explanation

- Gravity Head was cancelled out by \( h_1 (z_1 - z_2) \) and arm length difference of the manometer.
Find magnitude of $P$ to hold the gate.

$P_{h2o} = 9.8 \text{ kgs/m}^2$, width = $W$ meters, $+1$

For force on a curved surface, we can decompose into vertical and horizontal components.

By (2.26): $F_H = P_{cu} A$, where $P_{cu}$ is the hydrostatic pressure at the center of the projection of the curved surface on the vertical plane.

$P_{cu} = \rho \cdot h \cdot (2)(\frac{1}{2}) = (9790 \times 1) = 9790 \text{ Pa}$

$A$ is the area of projection of the curved surface on the vertical plane.

$A = (2)(w) = 2w \text{ m}^2$

$F_H = 19,580 \text{ wN} + 0.5$

The horizontal force acts at $Y'$ where $Y' = Y_c - \frac{I_{xx}}{Y_c A}$

- $Y_c$ is the position of the centroid of the projected surface on the vertical plane from the face surface.
- $I_{xx}$ is the area moment of inertia of the projected surface.
- $\phi$ is the angle of the projected surface with the horizontal.

$\phi = 90^\circ$ in this question.

$Y' = 1 - \frac{1}{12} \cdot \frac{Ah^2}{(1)(A)} = 0.666m + 0.5$
- Magnitude of Vertical component $F_v$

$= \text{Weight of fluid above the curved surface.}$

$= \gamma_{H_2O}V$

$F_v = \gamma_{H_2O} (\pi r^2)(\frac{1}{4})W$

$= (9790)(\pi)(2)^2(\frac{1}{4})W$

$= 30756 \text{ w N} + 05$

- $F_v$ acts on the center of gravity of the volume of fluid above the surface

接触点 of quarter circle:

$\frac{4r}{3\pi} = \frac{8}{3\pi} = 0.849 \text{ m} + 05$

Taking moment above C

$P = \frac{F_v}{0.66m} \times 0.543 \text{ m}$

$\sum M = 0: \quad -P(2) + FH(0.66) + Fv(0.54) = 0$

$P(2) = 13040 \text{ w} + 26111.8 \text{ w}$

$2P = 39152 \text{ w N}$

$\boxed{P = 19576 \text{ w N} + 1}$
Given water $SG = 1.0$, $h = 0$ for $SG = 1.0$

Buoyancy Force $F_B = \rho_{water} \cdot g \cdot V$

where $V$ is the displaced volume.

$FBD: \quad F_B = W$

$\sum F = 0 \quad F_B = W$

When $SG = 1$, let displaced volume $= V_0$, $X_{water} = X_0$

$F_B = \rho_{water} \cdot V_0$

$= X_{water} \cdot V_0$

$= X_0 \cdot V_0$

$\therefore W = X_0 \cdot V_0 \quad \Box$

For other fluid, where $SG \neq 1$

$W = \rho_{fluid} \cdot V_{fluid}$

$= (SG) \cdot X_0 \cdot (V_0 - Ah)$

$= (SG) \cdot X_0 \cdot V_0 - (SG) \cdot X_0 \cdot Ah$

By $\Box$

$W = (SG) \cdot W - (SG) \cdot X_0 \cdot Ah$

$(SG) \cdot X_0 \cdot Ah = (SG - 1) \cdot W$

$h = \frac{(SG - 1) \cdot W}{(SG) \cdot X_0}$

We can see in the figure, for $h > 0$, $V_{fluid} = V_0 - Ah$

Formulation $\Box + 2$

Steps $\Box + 1 + 1$ extra

Answer $\Box + 2$