1. Ping pong balls supported by turbulent jets, spinning baseballs, beachballs etc. (and turbulent eddies) move in response to the inertial vortex force $\mathbf{v} \times \mathbf{\omega}$, where $\mathbf{\omega}$ is curl $\mathbf{v}$. If the flow is irrotational, with $\mathbf{\omega}=0$, then the velocity can be expressed as the gradient of a velocity potential $\phi$; so that $\mathbf{v} = \nabla \phi$. What is the curl of $\nabla \phi$ (or $\nabla \rho$ etc.)?

(a) $\mathbf{v} \times \nabla \phi$ (b) the stream function (c) 0 (d) $a$ and $c$ (e) $a$ and $b$.

2. Use dimensional analysis to determine the Planck mass $m_p$ of particles and antiparticles, assuming the relevant dimensional parameters are the Planck constant $h = 1.05 \times 10^{-34}$ J s (kg m$^2$ s$^{-1}$), the speed of light $c = 3 \times 10^8$ m s$^{-1}$, and Newton’s gravitational constant $G = 6.7 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$. Do this by assuming the mass is proportional to $c$, $h$ and $G$ raised to unknown integer powers $d$, $e$ and $f$, so $m_p \sim c^d h^e G^f$. Write three equations for mass, length and time to determine the three unknown powers $d$, $e$ and $f$. What do you get?

(a) 1, 1 and 1 (b) 1, -1 and -1 (c) $\frac{1}{2}$, $-\frac{1}{2}$ and $-\frac{1}{2}$ (d) $\frac{1}{2}$, $\frac{1}{2}$ and $-\frac{1}{2}$ (e) 1, 1 and -1.

3. Continuum mechanics water salinity is defined by a limit process where the averaging volume is varied from molecular scales to turbulence scales. The salinity per unit volume at large scales is uncertain because salinity fluctuations have small diffusivity $D$ (compared to $v$) and are mixed by turbulence to Batchelor length scales $L_B \sim f(D, \nu) \sim D^{\frac{3}{4}} \gamma^{\frac{1}{4}}$, where salinity gradients are smoothed by diffusion. This is true not only for $v/D > 1$ (salt in water), but also for $v/D < 1$ (electron density in plasma). Find $a$ and $b$. $D = |D| = m^2 s^{-1}$. $\gamma = |\gamma| s^{-1}$.

(a) 1 and 1 (b) 1/2 and -1/2 (c) 1 and -1 (d) 1/2 and 1/2 (e) 1/3 and -1/3.

4. Continuum mechanics fluid velocity $\mathbf{v}$ is defined by a limit process. The momentum per unit mass $\mathbf{v}$ at large scales is uncertain because turbulent mixing of vorticity scrambles velocity gradients to length scales that depend on $\epsilon$ and $v$. Use dimensional analysis to find this scale. Set $L_\kappa \sim \epsilon^\alpha v^\beta$. What values of $a$ and $b$ are required to make this expression dimensionally homogeneous?

(a) $5/3$ and $1/3$ (b) $1/3$ and $-1/3$ (c) $\frac{1}{2}$ and $-\frac{1}{2}$ (d) $-1/4$ and $3/4$ (e) $-3/4$ and $+1/4$.

5. In the Navier Stokes equations the ratio of the inertial vortex force to the buoyancy force is known as the (a) Weber number (b) Ng number (c) Reynolds number (d) Sriram number (e) Froude number.

6. The divergence theorem is a special case of Gauss’ theorem, needed for deriving differential equations of fluid mechanics using control volumes that shrink sufficiently to validate the continuum hypothesis. It relates (a) stress tensors and velocity potentials (b) volume integrals and surface integrals (c) false (d) true (e) 42.

7. Stream functions are useful because lines of constant $\psi$ are streamlines. For a stream function to exist the flow must be (a) steady (b) irrotational (c) two dimensional (d) inviscid (e) all of the above plus incompressible.

8. The terms $u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$ can be written more compactly in vector form as

(a) $(\nabla \cdot \mathbf{v}) u$ (b) $(\nabla \cdot \mathbf{v}) u$ (c) $(\nabla \cdot \mathbf{u}) \mathbf{v}$ (d) $(\mathbf{u} \cdot \nabla) \mathbf{v}$ (e) $\nabla (u \cdot \mathbf{v})$

9. If the flow is incompressible, $\nabla \cdot \mathbf{v} =$

(a) unknown (b) $\rho$ (c) $u$ (d) $u \frac{\partial u}{\partial x}$ (e) 0.

10. For air at standard conditions, the flow can be considered incompressible if Mach number $M$ is less than (a) 1 (b) 0.5 (c) 0.3 (d) 0.1 (e) never.