For the incompressible Navier-Stokes Equations:

1. Which of these is nonlinear?
   (a) \( \rho \frac{\partial v}{\partial t} \)
   (b) \( -\nabla p \)
   (c) \( \bar{v} \cdot \nabla \bar{v} \)
   (d) \( \mu \nabla^2 v \)
   (e) all of them

2. For a 1-D flow in the y direction between 2 parallel, infinite vertical plates, separated by a width ‘h’ in the x direction, with constant pressure gradient ‘\( -\rho \mathbf{J} \)’ and body force of \( -g \rho \) in the y direction, the y momentum equation boils down to:
   (a) \( \frac{\partial p}{\partial x} = \frac{J - g - \rho \frac{2v_y^2}{\partial x^2}}{\partial y} \)
   (b) \( J = \rho \frac{\partial v_y}{\partial y} + \frac{\partial x}{\partial x} \)
   (c) \( \frac{\partial v_y}{\partial y} = J - g + \rho \frac{\partial x}{\partial x} \)
   (d) \( \frac{\partial v_y}{\partial y} = J - g + \rho \left( \frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} \right) \)
   (e) \( J = \rho g + \rho \left( \frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} \right) \)

3. A material or substantial derivative dB/dt of a scalar ‘B(x,t)’ is given as:
   (a) \( \partial B / \partial t \)
   (b) \( \rho v \cdot \nabla B \)
   (c) \( \rho \frac{\partial v_x}{\partial x} + \rho \frac{\partial v_y}{\partial y} \)
   (d) \( \rho \frac{\partial v_z}{\partial z} \)
   (e) \( \mu \nabla^2 v \)

4. The continuity equation is given by:
   (a) \( \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \)
   (b) conservation of mass for a system
   (c) divergence theorem
   (d) Cauchy’s rule
   (e) all but d

5. A simple 1-D diffusion process is given by:
   \( \frac{\partial \rho}{\partial t} = D \frac{\partial^2 \rho}{\partial x^2} \) (where ‘D’ is the diffusivity). The units of ‘D’ are:
   (a) m/s
   (b) m\(^2\)/s
   (c) m\(^3\)/s
   (d) \( m^2/s^2 \)
   (e) m\(^3\)/s
   (f) m\(^2\)/s

6. Viscous forces on surfaces are found by taking the scalar product of the outward pointing surface vector with the viscous stress tensor \( \mathbf{\tau} \). For Newtonian incompressible fluids, \( \mathbf{\tau} = 2 \mu \mathbf{e} \), where \( \mathbf{e} \) is the rate of strain tensor. The tensor is symmetric; so that \( e_{ij} = e_{ji} \) and \( e_{ij} = \frac{1}{2} \left[ \left( \partial v_i / \partial x_j + \partial v_j / \partial x_i \right) + \left( \partial v_i / \partial x_j - \partial v_j / \partial x_i \right) \right] \). Because \( \nabla \cdot \bar{v} = 0 = e_{ii} \) from incompressibility, then the sum of the principle values \( (\gamma_1, \gamma_2, \gamma_3) \) of the tensor must be zero. From the rate of change of the internal energy per unit volume derived in the math notes, the rate of heating by viscous friction of a turbulent fluid particle \( \rho \frac{Du}{Dt} = \mathbf{\tau} : \left( \nabla \bar{v} \right) = \rho \mathbf{\epsilon} \). Thus the units of \( \mathbf{\epsilon} \) are
   (a) m/s
   (b) m\(^3\)/s
   (c) m\(^2\)/s
   (d) m\(^2\)/s
   (e) m\(^3\)/s

Problems 7-10: The pressure drop in a pipe depends on diameter ‘d’, length ‘L’, velocity ‘v’, density ‘\( \rho \)’ and viscosity ‘\( \mu \)’

7. How many fundamental dimensions are involved and which ones?
   (a) 2; L and T
   (b) 3; L, T and M
   (c) 4; L, T, M and \( \theta \)
   (d) 3; L, M and \( \theta \)
   (e) 2; T and M

8. How many ‘pi groups’ will we have?
   (a) 2 (b) 3 (c) 4 (d) 5 (e) 1

9. Which of these is an appropriate selection for the recurring set of variables?
   (a) L, d and v (b) L, d and \( \rho \) (c) \( \mu \) and \( \rho \)
   (d) \( d, v \) and \( \rho \)
   (e) L and \( \mu \)

10. A simple pendulum with time period ‘T’ has mass ‘m’ , length ‘L’ and is subject to acceleration due to gravity ‘g’.
    From dimensional arguments, how might T depend on the other variables?
    (a) \( T \propto m(L/g)^{1/2} \)
    (b) \( T \propto gL \)
    (c) \( T \propto (L/g)^{1/2} \)
    (d) \( T \propto L/g \)
    (e) \( T \propto g^{1/2}L \)